1. Harvard Law School courses often have assigned seating to facilitate the “Socratic method.” Suppose that there are 100 first year Harvard Law students, and each takes two courses: Torts and Contracts. Both are held in the same lecture hall (which has 100 seats), and the seating is uniformly random and independent for the two courses.

(a) Find the probability that no one has the same seat for both courses (exactly; you should leave your answer as a sum).   
(b) Find a simple but accurate approximation to the probability that no one has the same seat for both courses.   
(c) Find a simple but accurate approximation to the probability that at least two students have the same seat for both courses.

1. There are 100 passengers lined up to board an airplane with 100 seats (with each seat assigned to one of the passengers). The first passenger in line crazily decides to sit in a randomly chosen seat (with all seats equally likely). Each subsequent passenger takes his or her assigned seat if available, and otherwise sits in a random available seat. What is the probability that the last passenger in line gets to sit in his or her assigned seat?

Solution

Let's solve these problems step by step.

### Problem 1: Harvard Law School Seating

#### Part (a): Probability that no one has the same seat for both courses

Given:

- There are 100 students.

- Each student is assigned a random seat in two different courses (Torts and Contracts).

- We need to find the probability that no student sits in the same seat for both courses.

This problem can be modeled using the concept of \*\*derangements\*\* (i.e., permutations where no element appears in its original position).

The probability that no one has the same seat for both courses is given by the ratio of the number of derangements (denoted as `!n`) to the total number of possible seatings:

\[

P(\text{no one has the same seat}) = \frac{!100}{100!}

\]

Where `!100` is the number of derangements of 100 objects.

The number of derangements is given by the sum:

\[

!n = n! \sum\_{k=0}^{n} \frac{(-1)^k}{k!}

\]

Thus, the probability is:

\[

P(\text{no one has the same seat}) = \sum\_{k=0}^{100} \frac{(-1)^k}{k!}

\]

#### Part (b): Approximation of the probability that no one has the same seat

The probability that no one has the same seat can be approximated using the fact that for large \(n\):

\[

P(\text{no one has the same seat}) \approx \frac{1}{e}

\]

Thus, for 100 students, the approximation is:

\[

P(\text{no one has the same seat}) \approx \frac{1}{e} \approx 0.3679

\]

#### Part (c): Approximation of the probability that at least two students have the same seat

The probability that at least two students have the same seat is the complement of the probability that no one has the same seat:

\[

P(\text{at least two students have the same seat}) \approx 1 - \frac{1}{e} \approx 1 - 0.3679 \approx 0.6321

\]

### Problem 2: Airplane Seating

Given:

- There are 100 passengers, each with an assigned seat.

- The first passenger sits in a randomly chosen seat.

- Each subsequent passenger sits in their assigned seat if available, otherwise sits in a random available seat.

- We need to find the probability that the last passenger (100th) sits in their assigned seat.

This problem is a well-known puzzle, and the solution is:

\[

P(\text{100th passenger sits in their assigned seat}) = \frac{1}{2}

\]

\*\*Explanation\*\*:

- The probability is independent of the number of passengers and remains 1/2 because, after the first passenger's random choice, the last passenger can either end up in their own seat or the first passenger's seat with equal probability.